

Markscheme

Specimen paper

Mathematics: applications and interpretation

Higher level

Paper 1

Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a correct **Method**.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- R** Marks awarded for clear **Reasoning**.
- AG** Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

*Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, e.g. **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (e.g. substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **M2**, **N3**, etc., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final **A1**. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct **FT** working shown, award **FT** marks as appropriate but do not award the final **A1** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	Award the final A1 (ignore the further working)
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final A1
3.	$\log a - \log b$	$\log(a - b)$	Do not award the final A1

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (e.g. probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin \theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Mis-copying of candidates’ own work does **not** constitute a misread, it is an error.
- The **MR** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for part-questions are indicated by **EITHER . . . OR**.

7 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation

The subject guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) discrete

A1

[1 mark]

(b)
$$\frac{24 + 60 + 3k + 40 + 15 + 6}{88 + k} = 2$$

M1A1

Note: Award **M1** for substitution into the formula for the mean, award **A1** for a correct equation.

attempt to solve their equation

(M1)

$k = 31$

A1

[4 marks]

- (c) systematic

A1

[1 mark]

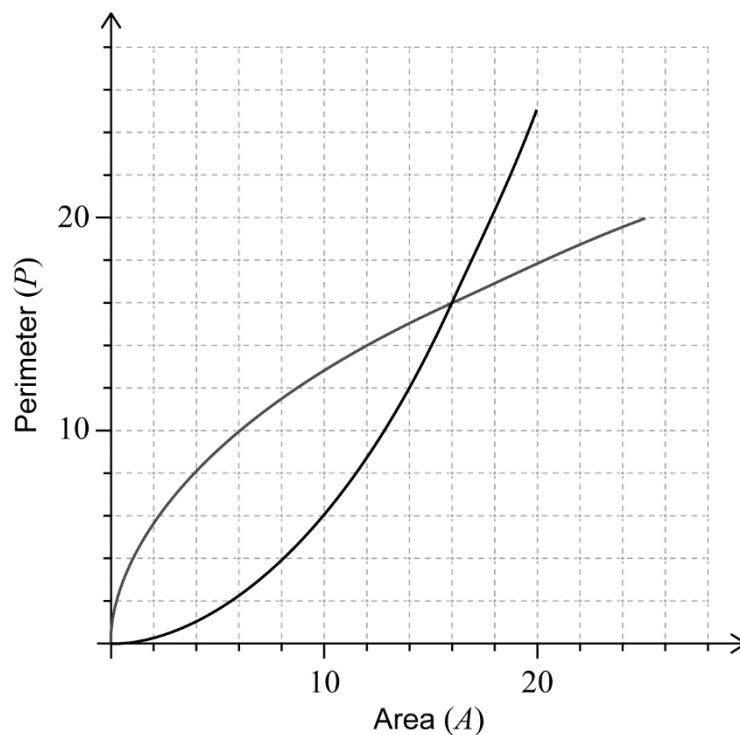
Total [6 marks]

2. (a) 20

A1

[1 mark]

- (b)



(M1)A1A1

Note: Award **(M1)** for reflection in the line $P = A$, award **A1** for endpoint at (20, 25), award **A1** for passing through (16, 16).

[3 marks]

- (c) when the perimeter is 8, the area is 4

A1

[1 mark]

Total [5 marks]

3. (a) (i) 1750 A1

(ii) $1350 + 400 (1.25)^{-5}$ (M1)

= 1480 A1

Note: Accept 1481.

[3 marks]

(b) $1400 = 1350 + 400 (1.25)^{-t}$ (M1)

9.32 (days (9.31885...) (days)) A1
[2 marks]

(c) 1350 A1

Note: Accept 1351 as a valid interpretation of the model as $P=1350$ is an asymptote.

[1 mark]

Total [6 marks]

4. (a) $\frac{3-1}{7-3}$ (M1)

= 0.5 A1
[2 marks]

(b) $y-2 = -2(x-5)$ (A1)(M1)

Note: Award (A1) for their -2 seen, award (M1) for the correct substitution of (5, 2) and their normal gradient in equation of a line.

$2x + y - 12 = 0$ A1
[3 marks]

(c) every point in the cell is closer to E than any other snow shelter A1
[1 mark]

Total [6 marks]

5. (a) $\frac{50 \times \pi}{180} = 0.873 \text{ (0.872664...)}$ **A1**

[1 mark]

(b) volume = $240 \left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664... \right)$ **M1M1M1**

Note: Award **M1** $240 \times \text{area}$, award **M1** for correctly substituting area sector formula, award **M1** for subtraction of the angles or their areas.

= 45800 (= 45811.96071) **A1**

[4 marks]

Total [5 marks]

6. (a) $\frac{4}{18} \left(\frac{2}{9} \right)$ **A1**

[1 mark]

(b) $-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \dots + 5 \times \frac{7}{18}$ **(M1)**

Note: Award **(M1)** for their correct substitution into the formula for expected value.

= $1.83 \left(\frac{33}{18}, 1.83333... \right)$ **A1**

[2 marks]

(c) $2 \times \frac{1}{18} \times \frac{3}{18}$ **(M1)(M1)**

Note: Award **(M1)** for $\frac{1}{18} \times \frac{3}{18}$, award **(M1)** for multiplying their product by 2.

= $\frac{1}{54} \left(\frac{6}{324}, 0.0185185..., 1.85\% \right)$ **A1**

[3 marks]

Total [6 marks]

7. (a) $E = 5(2 \sin t)^2 (= 20 \sin^2 t)$ **A1**
[1mark]
- (b) $\frac{dE}{dt} = 40 \sin t \cos t$ **(M1)A1**
[2 marks]
- (c) $t = 0.126$ **(M1)A1**
[2 marks]
- Total [5 marks]**

8. (a) $\frac{\sin \hat{CAB}}{6} = \frac{\sin 15^\circ}{4.5}$ **(M1)(A1)**

$\hat{CAB} = 20.2^\circ$ (20.187415...) **A1**

Note: Award **(M1)** for substituted sine rule formula and award **(A1)** for correct substitutions.

[3 marks]

(b) $\hat{CBD} = 20.2 + 15 = 35.2^\circ$
(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741...^\circ = \frac{1.8}{BX}$ **(M1)**

Note: Award **A1** for their correct angle \hat{CBD} . Award **M1** for correctly substituted trigonometric formula.

$BX = 2.55285...$ **A1**

$5 - 2.55285...$ **(M1)**

$= 2.45 \text{ (m)} \text{ (2.44714...)}$ **A1**
[5 marks]

Total [8 marks]

9. (a) $s_{n-1} = \sqrt{\frac{10}{9}} \times 0.0196 = 0.02066\dots$

(M1)A1

[2 marks]

(b) (1.463, 1.493)

(M1)A1

Note: If s_n used answer is (1.464, 1.492), award **M1A0**.

[2 marks]

(c) 95% of the time these results would be produced by a population with mean of less than 1.5 kg, so it is likely the mean weight is less than 1.5 kg

R1

[1 mark]

Total [5 marks]

10. let T be the time to serve both customers and T_i the time to serve the i th customer

assuming independence of T_1 and T_2

R1

T is normally distributed and $T = T_1 + T_2$

(M1)

$E(T) = 1.5 + 1.5 = 3$

A1

$\text{Var}(T) = 0.4^2 + 0.4^2 = 0.32$

M1A1

$P(T < 4) = 0.961$

A1

Total [6 marks]

11. (a) $15 \times 0 + 2d + 4 = 0$
 $d = -2$

(M1)

A1

[2 marks]

(b) $a \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$

(M1)

$= a \begin{pmatrix} 10 \\ 15 \\ 30 \end{pmatrix} \left(= 5a \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \right)$

A1

magnitude is $5a\sqrt{2^2 + 3^2 + 6^2} = 14$

M1

$a = \frac{14}{35} \quad (= 0.4)$

A1

[4 marks]

Total [6 marks]

12. (Model A)

$$R = 3pe^{-0.5p}$$

M1

predicted values

p	R
1	1.8196
2	2.2073
3	2.0082

(A1)

$$SS_{res} = (1.8196 - 1.5)^2 + (2.2073 - 1.8)^2 + (2.0082 - 1.5)^2$$

(M1)

$$= 0.5263\dots$$

A1

(Model B)

$$R = 2.5pe^{-0.6p}$$

predicted values

p	R
1	1.372
2	1.506
3	1.2397

(A1)

$$SS_{res} = 0.170576\dots$$

A1

chose model B

A1

Note: Method marks can be awarded if seen for either model A or model B. Award final **A1** if it is a correct deduction from their calculated values for A and B.

Total [7 marks]

13. (a) $\frac{dy}{dx} = \frac{16-20}{24-20}$
 $= -1$

M1

A1

[2 marks]

(b) asymptote of trajectory along $r = k \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

M1A1

Note: Award **M1A0** if asymptote along $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

trajectory begins at (8, 10) with negative gradient

A1A1

[4 marks]

Total [6 marks]

14. (a) (i) $a = 33$

A1

(ii) $\frac{1}{\sqrt[3]{0.08}} = 2.32$

M1A1

[3 marks]

(b) volume within outer dome

$$\frac{2}{3}\pi \times 16^3 + \pi \times 16^2 \times 17 = 22\,250.85$$

M1A1

volume within inner dome

$$\pi \int_0^{33} \left(\frac{33-y}{0.08} \right)^{\frac{2}{3}} dy = 3446.92$$

M1A1

volume between = $22\,250.85 - 3446.92 = 18\,803.93 \text{ m}^3$

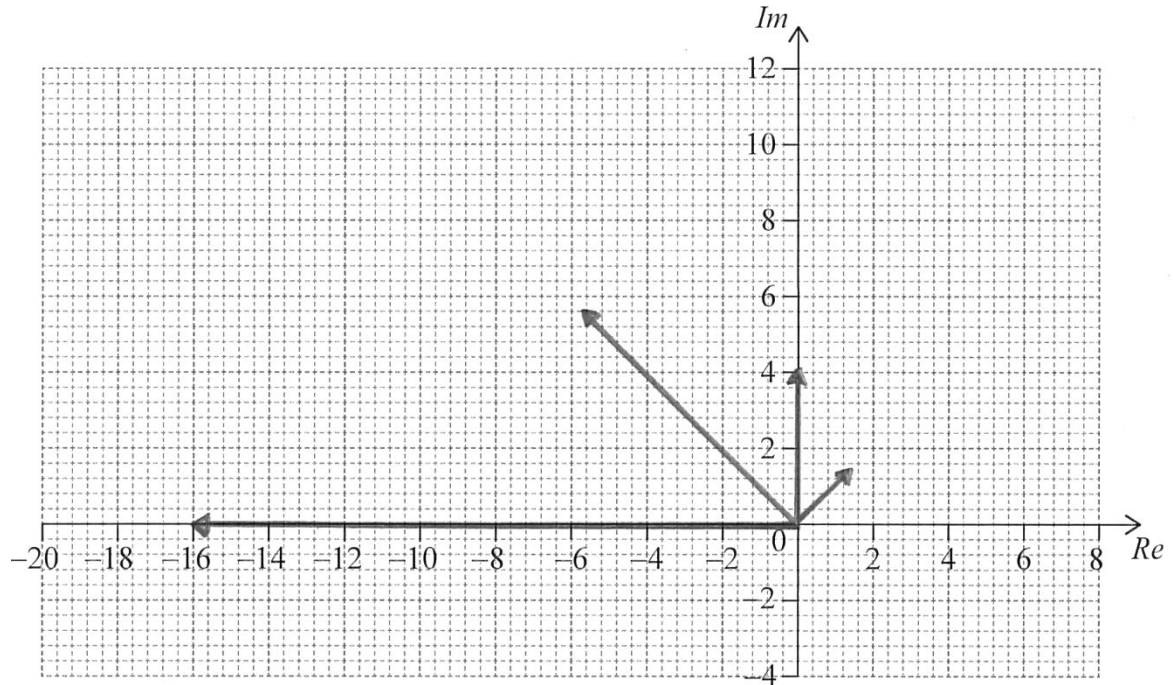
A1

[5 marks]

Total [8 marks]

15. (a) (i) $4e^{\frac{\pi}{2}i}, 8e^{\frac{3\pi}{4}i}, 16e^{\pi i} \left(= 4i, -4\sqrt{2} + 4\sqrt{2}i, -16\right)$ **(M1)A1**

(ii)



A3

Note: Award **A1** for correct arguments, award **A1** for $4i$ and -16 clearly indicated, award **A1** for $|w| < 4$ and $4 < |w^3| < 16$.

[5 marks]

(b) $2^2 + 1^2 = a^2$ **M1**

$a = \sqrt{5} \left(= 2.24\right)$ **A1**

[2 marks]

Total [7 marks]

16. (a) $H_0 : m = 3, H_1 : m < 3$

A1

Note: Accept equivalent statements in words.

[1 mark]

- (b) (let X be the number of fish caught)
 $P(X \leq 1 | m = 3) = 0.199$

M1A1

[2 marks]

- (c) $P(X \geq 2 | m = 2.5) (= 1 - P(X \leq 1 | m = 2.5))$

M1A1

Note: Award **M1** for using $m = 2.5$ to evaluate a probability,
 award **A1** for also having $X \geq 2$.

$$= 0.713$$

A1

[3 marks]

Total [6 marks]

17. (a) $P(X = 8)$

(M1)

Note: Award **(M1)** for evidence of recognizing binomial probability.

eg, $P(X = 8), X \sim B\left(20, \frac{6}{15}\right)$.

$$= 0.180 \text{ (0.179705...)}$$

A1

[2 marks]

- (b) let x be the number of male students

recognize that probability of selecting a male is equal to $\frac{x}{80}$

(A1)

$$\left(\text{set up equation } {}^{20}C_8 \left(\frac{x}{80} \right)^8 \left(\frac{80-x}{80} \right)^{12} = \right) 0.153357$$

(M1)

number of male students = 37

(M1)A1

Note: Award **(M1)A0** for 27.

[4 marks]

Total [6 marks]

18. $\log A = x \log B + y \log C + \log k$ **(M1)**

$$\log 5.74 = x \log 2.1 + y \log 3.4 + \log k$$

$$\log 2.88 = x \log 1.5 + y \log 2.4 + \log k$$

$$\log 0.980 = x \log 0.8 + y \log 1.9 + \log k$$
 M1A1

Note: Allow any consistent base, allow numerical equivalents.

attempting to solve their system of equations **(M1)**

$$x = 1.53, y = 0.505$$
 A1

$$k = 0.997$$
 A1

Total [6 marks]